PHYS 101 – General Physics I Midterm Exam

Duration: 120 minutes Saturday, 22 October 2022; 09:30

1. A particle, moving on a horizontal, frictionless xy plane, is tied to the origin with a massless, inextensible string of length L. At time $t = 0$, its position vector is given as $\vec{\mathbf{r}}(t=0) = L\hat{\mathbf{i}}$, and its velocity vector is given as $\vec{\mathbf{v}}(t=0) = v_0\hat{\mathbf{j}}$.

(a) (7 Pts.) Find the position vector $\vec{\mathbf{r}}(t)$ of the particle as a function of time.

(b) (6 Pts.) Find the acceleration vector $\vec{\mathbf{a}}(t)$ of the particle as a function of time.

At time $t = \frac{5\pi L}{3v_0}$ the string connecting the particle to the origin snaps, subsequently, no forces act on the particle.

(c) (6 Pts.) Find the velocity vector of the particle at all later times.

(d) (6 Pts.) When and where does the particle cross the x axis in terms of v_0 and L?

Solution:

Particle moving around a circle of radius L with constant speed v_0 means uniform circular motion. Given

$\vec{\mathbf{r}}(0) = L\,\hat{\mathbf{i}}$, $\vec{\mathbf{v}}(0) = v_0\,\hat{\mathbf{j}}$, $\omega = v_0/L$, so we have

$$\vec{\mathbf{r}}(t) = L\cos\left(\frac{v_0 t}{L}\right)\hat{\mathbf{i}} + L\sin\left(\frac{v_0 t}{L}\right)\hat{\mathbf{j}}\,.$$

Therefore

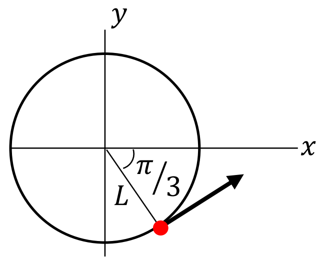
$$\frac{d\vec{\mathbf{r}}(t)}{dt} = \vec{\mathbf{v}}(t) = -v_0\sin\left(\frac{v_0 t}{L}\right)\hat{\mathbf{i}} + v_0\cos\left(\frac{v_0 t}{L}\right)\hat{\mathbf{j}}\,,$$

and

$$\frac{d\vec{\mathbf{v}}(t)}{dt} = \vec{\mathbf{a}}(t) = -\left(\frac{v_0^2}{L}\right)\cos\left(\frac{v_0 t}{L}\right)\hat{\mathbf{i}} - \left(\frac{v_0^2}{L}\right)\sin\left(\frac{v_0 t}{L}\right)\hat{\mathbf{j}} = -\left(\frac{v_0^2}{L}\right)\left[\cos\left(\frac{v_0 t}{L}\right)\hat{\mathbf{i}} + \sin\left(\frac{v_0 t}{L}\right)\hat{\mathbf{j}}\right].$$

When the string snaps

$$\theta = \frac{v_0}{L}\frac{5\pi L}{3v_0} = \frac{5\pi}{3} \quad\to\quad \cos\left(\frac{5\pi}{3}\right) = 1/2\,, \qquad \sin\left(\frac{5\pi}{3}\right) = -\sqrt{3}/2\,.$$



$$\vec{\mathbf{r}}\left(\frac{5\pi L}{3v_0}\right) = \frac{L}{2}\,\hat{\mathbf{i}} - \frac{\sqrt{3}L}{2}\,\hat{\mathbf{j}}\,, \qquad \vec{\mathbf{v}}\left(\frac{5\pi L}{3v_0}\right) = \frac{\sqrt{3}v_0}{2}\,\hat{\mathbf{i}} + \frac{v_0}{2}\,\hat{\mathbf{j}}\,.$$

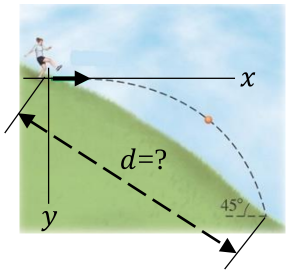
Therefore, assuming that the string snaps at time $t' = 0$ for the new initial value problem (i.e., $t' = t - \frac{5\pi L}{3v_0}$), we have

$$\vec{\mathbf{r}}(t) = \left(\frac{L}{2}\,\hat{\mathbf{i}} - \frac{\sqrt{3}L}{2}\,\hat{\mathbf{j}}\right) + \left(\frac{\sqrt{3}v_0}{2}\,\hat{\mathbf{i}} + \frac{v_0}{2}\,\hat{\mathbf{j}}\right)t' = \left(\frac{L}{2} + \frac{\sqrt{3}v_0}{2}t'\right)\hat{\mathbf{i}} + \left(-\frac{\sqrt{3}L}{2} + \frac{v_0}{2}t'\right)\hat{\mathbf{j}}\,.$$

Crossing the x axis means

$$y = 0 \quad\to\quad t' = \sqrt{3}\frac{L}{v_0}\,,\ x\left(\sqrt{3}\frac{L}{v_0}\right) = 2L\,.$$

2. A rock is kicked horizontally with initial speed v_0 from a hill with a $\pi/4$ slope. Use the coordinate system indicated in the figure to answer the following questions. (Gravitational acceleration is g.)



(a) (7 Pts.) How long does it take for the rock to hit the ground?

(b) (9 Pts.) How far away (distance $d=$? along the hill) does the ball hit the ground?

(c) (9 Pts.) What is the tangent of the angle between its velocity vector and the horizontal axis when it hits the ground?

Solution:

$$x(t) = v_0 t\,, \qquad y(t) = \frac{1}{2}\mathrm{g}t^2$$

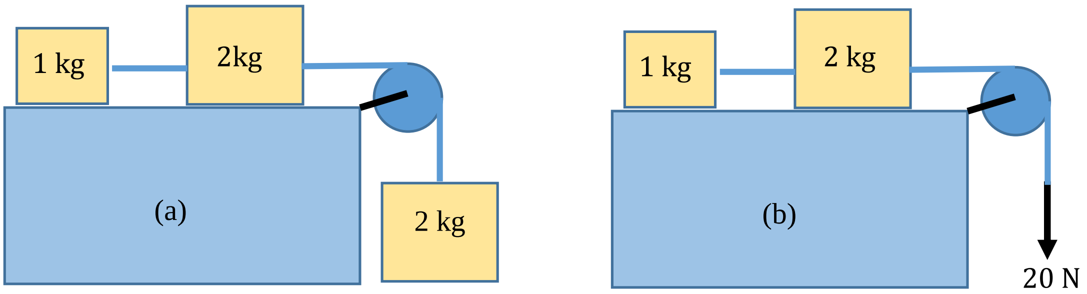
When the ball hits the ground at time T, we have $y = x$, So

$$\frac{1}{2}\mathrm{g}T^2 = v_0 T \quad \rightarrow \quad T = 0\,, \qquad T = \frac{2v_0}{\mathrm{g}}\,.$$

$$x(T) = y(T) = \frac{2v_0^2}{\mathrm{g}} \quad \rightarrow \quad d = 2\sqrt{2}\left(\frac{v_0^2}{\mathrm{g}}\right).$$

$$\vec{\mathbf{v}}(T) = v_0\,\hat{\mathbf{i}} + \mathrm{g}T\,\hat{\mathbf{j}}\,, \qquad \vec{\mathbf{v}}(T) = v_0\,\hat{\mathbf{i}} + 2v_0\,\hat{\mathbf{j}} \quad \rightarrow \quad \tan\varphi = 2\,.$$

3.



Three masses are tied to each other with massless, inextensible cords over a frictionless pulley as shown in figure (a). The table surface is horizontal and frictionless. The value of gravitational acceleration is given as 10 $\mathrm{m/s^2}$.

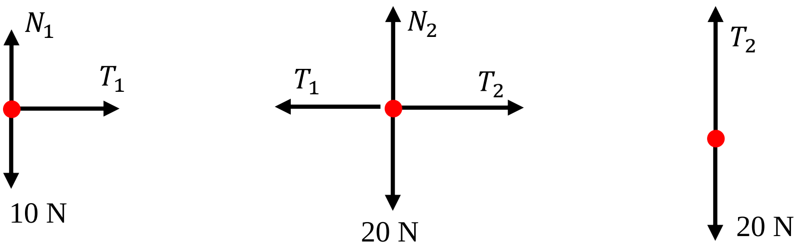
(a) (6 Pts.) Draw free body diagrams for all masses.

(b) (7 Pts.) Find the acceleration of the system.

(c) (7 Pts.) What is the tension in the cord connecting the 1kg mass to the 2 kg mass?

(d) (5 Pts.) Now consider the arrangement in figure (b), where one of the 2 kg masses is removed, but the cord hanging from the pulley is pulled down with 20 N of force. Does the tension in the cord connecting the 1 kg mass to the 2 kg mass increase, decrease, or stays the same? Explain your answer.

Solution:



$$T_1 = a\,, \qquad T_2 - T_1 = 2a\,, \qquad 20 - T_2 = 2a \quad \rightarrow \quad T_2 = 3a \quad \rightarrow \quad 5a = 20 \quad \rightarrow \quad a = 4\ \mathrm{m/s^2}\,, T_1 = 4\ \mathrm{N}\,.$$

For part (d):

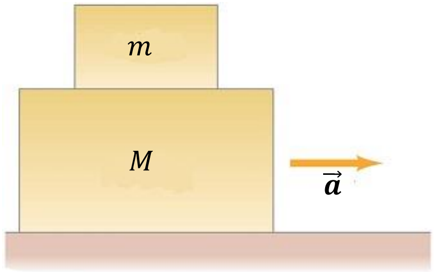
$$T'_2 = 20\ \mathrm{N}\,, \qquad T'_1 = a', \qquad T'_2 - T'_1 = 2a' \quad \rightarrow \quad a' = \frac{20}{3}\ \mathrm{m/s^2} \quad \rightarrow \quad T'_1 = \frac{20}{3}\ \mathrm{N}\,.$$

Since

$$\frac{20}{3}\ \mathrm{N} > 4\ \mathrm{N}\,,$$

the tension increases.

4. A block with mass m is stacked on top of another block with mass M, which is accelerating along a horizontal table with acceleration a, as shown in the figure. Let $\mu_k = \mu_s = \mu$ between the blocks, and assume that the table is frictionless.



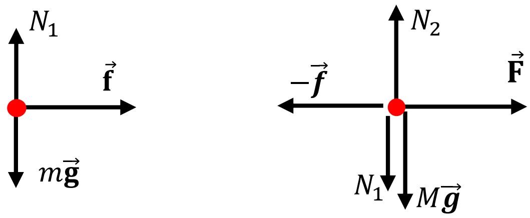
(a) (4 Pts.) Draw a free body diagram for both blocks.

(b) (5 Pts.) What minimum coefficient of friction μ_s between the two blocks will prevent the top block from sliding off?

(c) (8 Pts.) If μ_s is only half this minimum value, what is the acceleration of the top block with respect to the table, and with respect to the bottom block?

(d) (8 Pts.) What is the force that must be applied to the bottom block in parts (b) and (c)?

Solution: (a)



(b) Both blocks move with the same acceleration a.

Considering the free body diagram of the top block, we have

$$f_s = ma\,, \qquad N_1 = mg\,, \qquad f_s \le \mu_s N_1 \quad \to \quad f_s \le \mu_s mg \quad \to \quad \mu_s \ge \frac{a}{g} \quad \to \quad \mu_{s\,\min} = \frac{a}{g}.$$

(c) In this case $\mu = \mu_k = a/2g$, meaning that the acceleration of the top block is less than a. Now we have

$$f_k = \mu_k mg = \left(\frac{a}{2g}\right) mg = \frac{1}{2} ma\,, \qquad f_k = ma_1 \quad \to \quad ma_1 = \frac{1}{2} ma \quad \to \quad a_1 = \frac{a}{2}.$$

relative to the ground. Acceleration of the top block relative to the bottom block is

$$a_{12} = a_1 - a = -\frac{a}{2}.$$

(d) We consider the free body diagram of the bottom block.

$$F - f = Ma \quad \to \quad F = f + Ma\,.$$

$$F = (m+M)a \ \text{ for part (b)}, \qquad \text{and } F = \left(\frac{m}{2} + M\right) a \ \text{ for part (c)}\,.$$